$$\frac{\left(x - \frac{2z}{\tan \alpha}\right)^2}{\frac{8z^2}{\sin^2 \alpha}} + \frac{y^2}{8z^2} = 1. \quad (46)$$

This is seen to be the standard Cartesian form for an ellipse.

As a practical matter it is of interest to know the distance from the antenna base at B to the near edge of the first zone d_N, to the far edge d_F, and the maximum width w, or the minor axis, of the zone. These distances, increased suitably for the finite dimensions of a practical antenna, determine the extent and location of the ground in front of the antenna which must be flat and free of obstructions for effective lobe formation. From (46) when $y=0$,

$$x = \frac{2z}{\tan \alpha}\left(1 \pm \frac{\sqrt{2}}{\cos \alpha}\right), \quad (47)$$

so that,

$$d_N = \frac{z}{\tan \alpha}\left(3 - \frac{2\sqrt{2}}{\cos \alpha}\right), \quad (48)$$

$$d_F = \frac{z}{\tan \alpha}\left(3 + \frac{2\sqrt{2}}{\cos \alpha}\right), \quad (49)$$

and the maximum width of the zone is:

$$w = 4\sqrt{2}z. \quad (50)$$

The effect of the curvature of the earth on the first Fresnel zone will be appreciable only if the angle of arrival or departure, α, is very small. The qualitative result of curvature is to cause the elliptical area to be reduced in size and to be altered into an egg shape elongated with the broad end near the antenna. The quantity d_N is almost unaltered, whereas d is somewhat reduced, and d_F is considerably reduced. The maximum width will be slightly reduced and will occur somewhat nearer the antenna. An additional effect of some practical importance is the reduction in the height, z, of the antenna required for the first lobe maximum to be formed at a specified angle α. For the plane-earth case this height was given by (43).

Curves of z and d have been derived, and are plotted in Figs. 65, 66, and 67 as a function of α for plane earth and for the curved earth, for three fixed values of λ corresponding to frequencies of 30, 40, and 50 Mc/s. Some compensation for the effects of tropospheric refraction has been introduced for curved-earth case. Fig. 68 (next page) gives values of d_F as a function of α for the three sample frequencies for a plane earth according to (49). When α is greater than two or three degrees the values for a curved earth will be smaller by a negligible amount.

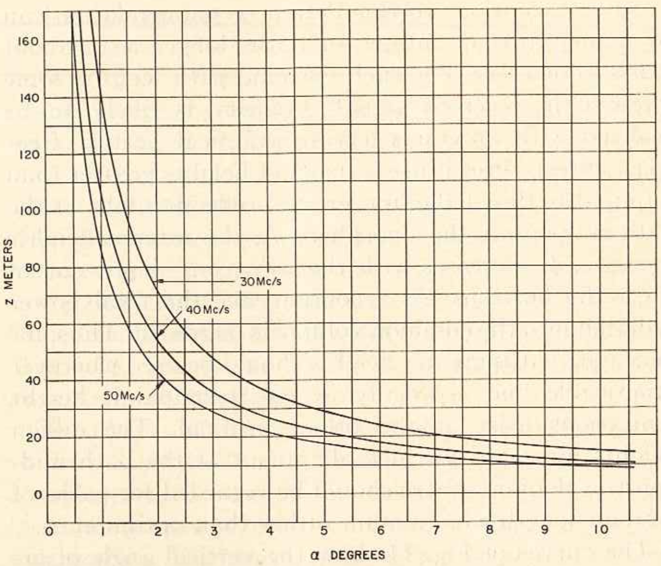

Fig. 65—Antenna heights for aligning first ground-reflection lobe maximum at indicated elevation angle, for horizontal polarization and a plane earth.

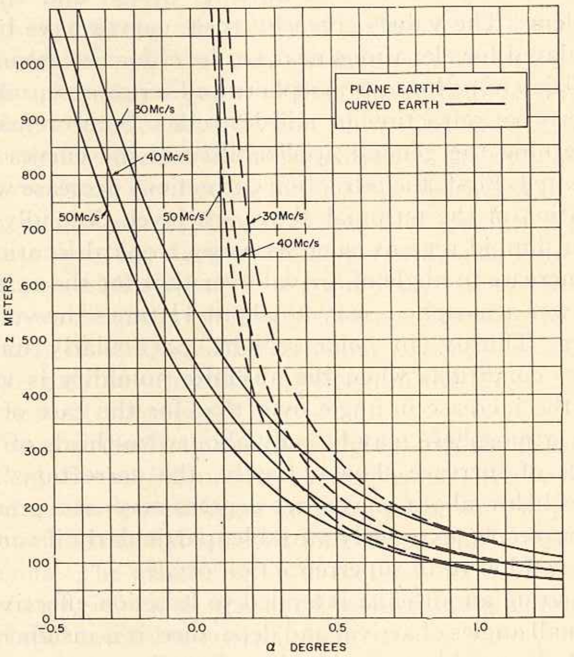

Fig. 66—Antenna heights for aligning first ground-reflection lobe maximum at indicated low elevation angle for horizontal polarization, for plane and curved earth.

When α is less than two or three degrees the curves shown give values of d_F substantially greater than the actual values for a curved earth, and provided they can be satisfied in practice no difficulties can be expected to result from their use.

It is of practical importance to know what sort of departures from the ideal first Fresnel zone may be tolerated. For a well-developed first ground-reflection lobe the entire first zone at least, should be flat, and the horizon from every part of it should subtend less than

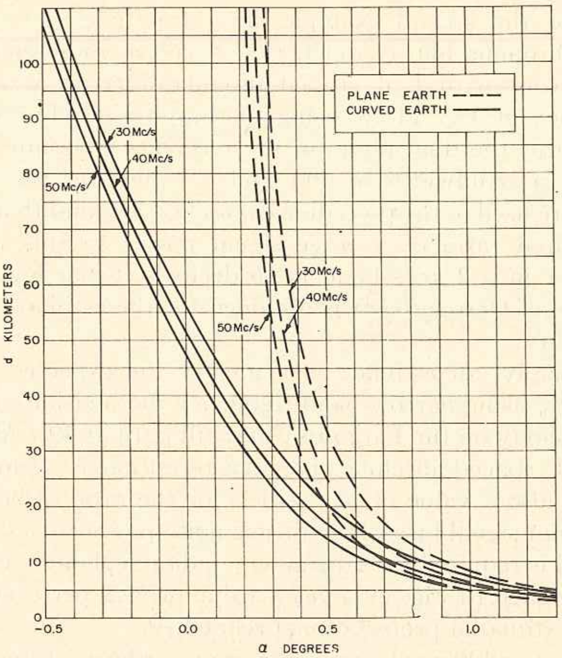

Fig. 67—Distance to ground-reflection point from antenna having first ground-reflection lobe maximum at indicated low elevation angle for horizontal polarization, for plane and curved path.

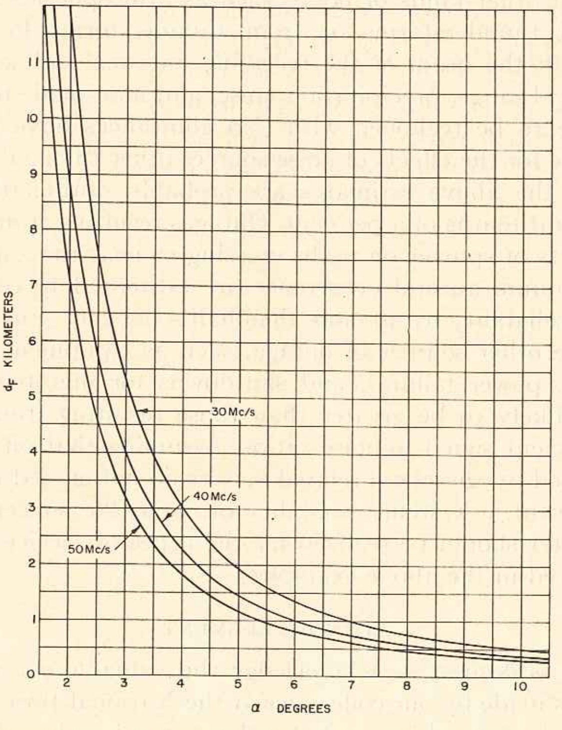

Fig. 68—Distance to far edge of first Fresnel zone from antenna having first ground-reflection lobe maximum at indicated elevation angle for horizontal polarization and plane earth.

the angle α. Norton and Omberg,[35] applying Rayleigh's criterion, state that irregularities in the terrain in the first zone should have a departure from ideal smoothness of not more than one-fourth of the antenna height. As a practical matter, water surfaces make excellent first Fresnel zones, particularly for small values of α.

The ray treatment leading to the results presented above for the curved earth should not be relied upon for values of α less than about 0 degree at which the horizon, for the heights involved, is actually below minus half a degree. For one thing the divergence factor begins to reduce the effectiveness of the ground-reflection in the formation of the lower lobes, though this may be offset by other considerations, as discussed for example by Burrows and Attwood.[36]

Modulation Techniques

In the development of systems utilizing vhf ionospheric propagation it is important that consideration be given to the special behavior characteristics of the received signals in order to realize a system capable of minimizing or eliminating the undesirable transmission effects produced by reflections from meteoric ionization and the fading and multipath characteristics of the normally received signals. Considerations of path geometry and meteor velocities indicate that the difference between the transmitted frequency and the Doppler frequencies should not be greater than about 6 kc/s for the Cedar Rapids to Sterling path at 49.8 Mc/s. This difference in frequency is, for a particular meteoric event, directly proportional to the transmitted frequency.

Estimated System Performance

A considerable body of signal intensity and noise intensity data has been acquired in the experimental program. It is possible, using these data, to estimate the expected reliability of a system employing a typical type of service such as single-channel radioteletype transmission. System reliability estimates are made for several assumed types of service.

Montgomery[37,38] has studied the behavior of several types of modulation for narrow-band transmission of binary-coded messages in the presence of fluctuation noise and has considered the effects of diversity action in the reception of narrow-band frequency-shift transmission. His results will be used for estimating ratios of average signal power to rms noise power required to establish a specified system performance for radiotele-

[35] K. A. Norton and A. C. Omberg, "The maximum range of a radar set," Proc. IRE, vol. 35, pp. 4–24; January, 1947.

[36] C. R. Burrows and S. S. Attwood (Eds.), "Radio Wave Propagation," Academic Press, Inc., New York, N. Y., pp. 80–81, 119–120; 1949.

[37] G. F. Montgomery, "A comparison of amplitude and angle modulation for narrow-band communication of binary-coded messages in fluctuation noise," Proc. IRE, vol. 42, pp. 447–454; February, 1954.

[38] G. F. Montgomery, "Message error in diversity frequency-shift reception," Proc. IRE, vol. 42, pp. 1184–1187; July, 1954.